Numerical Lidar - modelling lidar volume-averaging

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Motivation

Increasing application of wind lidars in ...

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Research & Development

- Rotor flows
- Complex terrain (mountains, forests ...)
- Wind farms

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Industry

- Site assessment on- and offshore
- Inflow measurements

Numerical lidar

Lidar measurement "errors"

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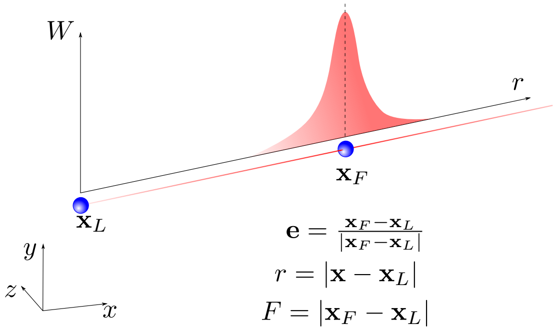
Aim

- Integrate volume-averaging in numerical studies
- Balance accuracy and computational cost

Lidar volume-averaging



$$\mathbf{e} = \frac{\mathbf{x}_F - \mathbf{x}_L}{|\mathbf{x}_F - \mathbf{x}_L|}$$

$$r = |\mathbf{x} - \mathbf{x}_L|$$

$$F = |\mathbf{x}_F - \mathbf{x}_L|$$

Line-of-sight velocity

$$v_{los}(\mathbf{x}_F) = -\int_0^\infty \mathbf{e}(\mathbf{x}_F, \mathbf{x}_L) \cdot \mathbf{V}(r) W(r)\, \mathrm{d}r$$

Weighting functions

Line-of-sight velocity

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Continous-wave (CW)

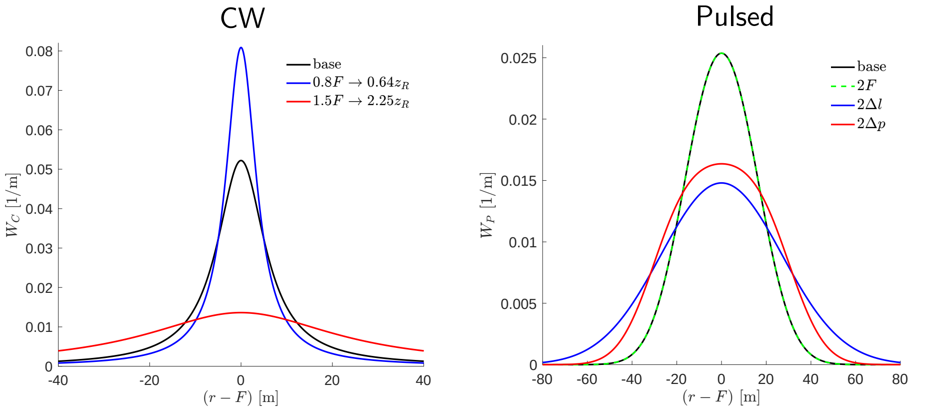
$$W_C(r) = \frac{1}{\pi} \frac{z_R}{z_R^2 + (r-F)^2} \quad \text{with} \quad z_R = \frac{\lambda F^2}{\pi {\alpha_0}^2}$$

Pulsed

$$W_P(r) = \frac{1}{2\Delta p} \left\{ \mathrm{Erf}\left[\frac{(r-F) + {}^{\Delta p}/_2}{r_p} \right] - \mathrm{Erf}\left[\frac{(r-F) - {}^{\Delta p}/_2}{r_p} \right] \right\}$$

$$\text{with} \quad \mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) \, \mathrm{d}t \quad \text{and} \quad r_p = \frac{\Delta l}{2\sqrt{\ln(2)}}$$

Weighting functions



Discretisation of weighting functions

$$v_{los}(\mathbf{x_F}) = \sum_{i=1}^{n_W} \mathbf{e}(\mathbf{x}_F, \mathbf{x}_L) \cdot \mathbf{V}(r_i) W(r_i)$$

minimise n_w to decrease computational cost from interpolation

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$$\Delta A = \int_{r_i}^{r_{i+1}} W(r)\, \mathrm{d}r = const$$

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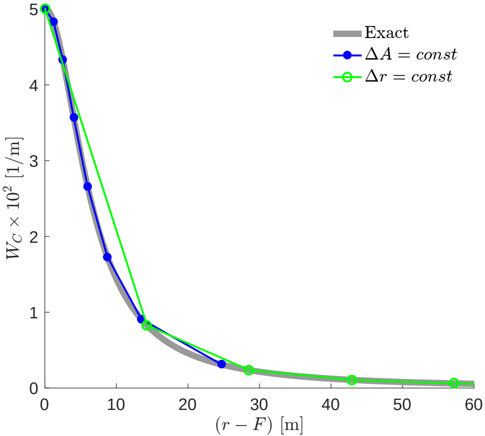


Wake flow

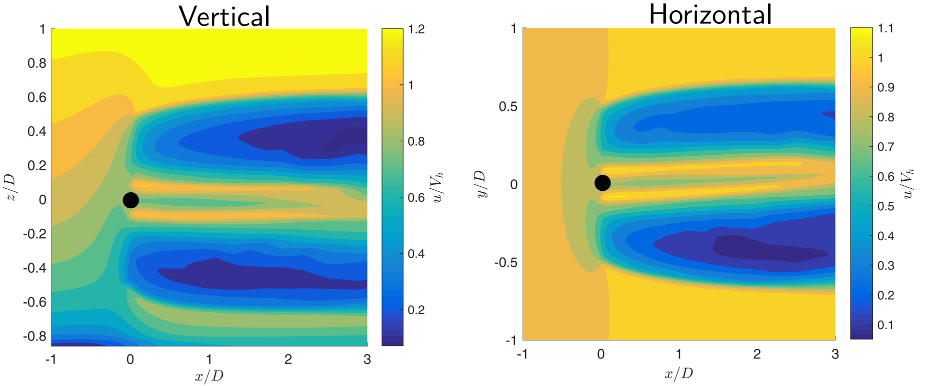
Numerical method

- EllipSys3D: DES with $k-\omega$ SST and a SGS model
- Actuator line: Siemens SWT 93-2.3 - rotor diameter: 93 m - hub height: 80 m - 40 actuator points
- Box domain $25R$ side length - Δx: R/32 - well cell height: 0.05 m

Numerical setup

- $V_{h,\infty} = 8$ m/s
- Power law with 0.3 shear exponent
- $C_T = 0.81$
- A single lidar positioned at rotor centre

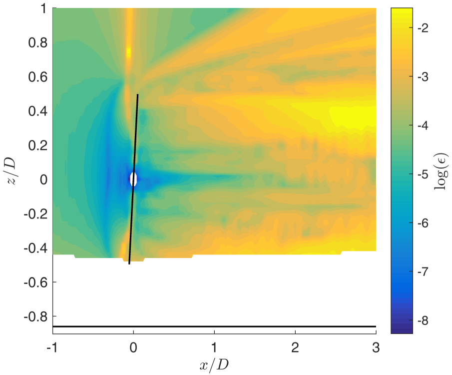
Wake flow

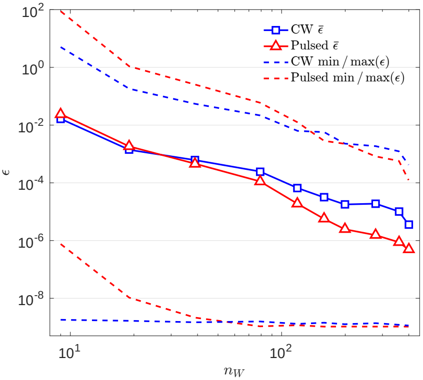


Normalised contours of the time-averaged streamwise velocity

Convergence of beam discretisation

$$\epsilon^i = \frac{v_{los}^i - v_{los}^{i\text{-}1}}{v_{los}^{i\text{-}1}}$$



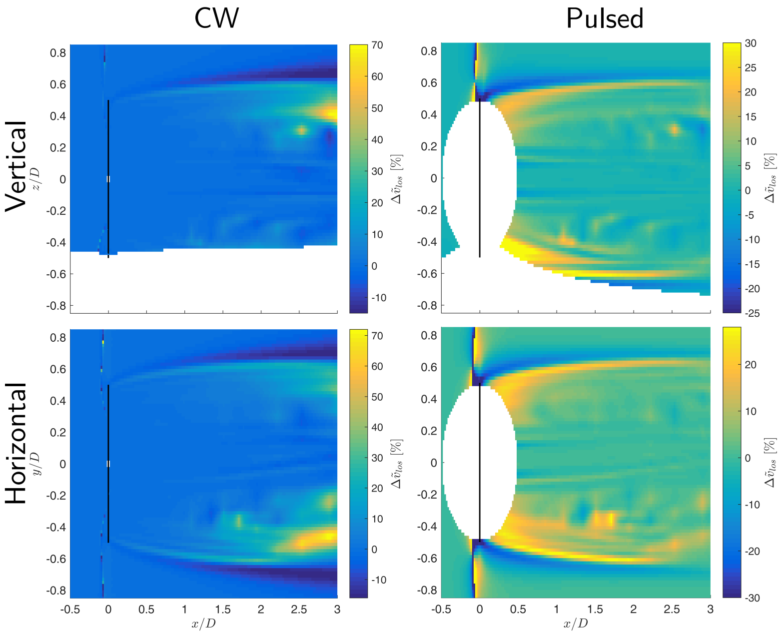


Contours of v_{los} residual for the CW lidar with $n_W = 19$.

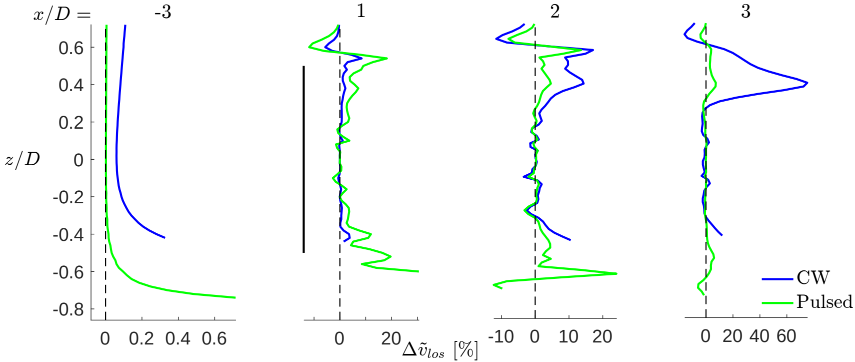
Effect of volume-averaging

$$\Delta \tilde{v}_{los} = \frac{v_{los}^{\text{lidar}} - v_{los}^{\text{point}}}{v_{los}^{\text{point}}}$$

Effect of volume-averaging

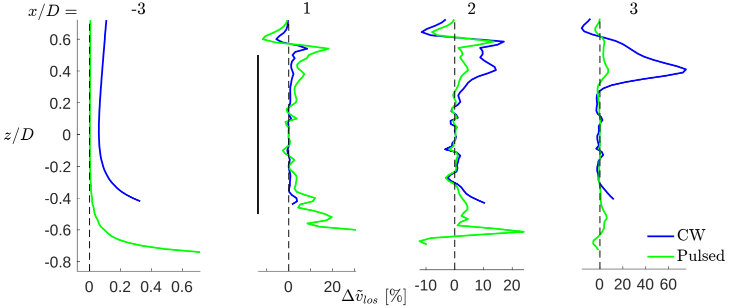


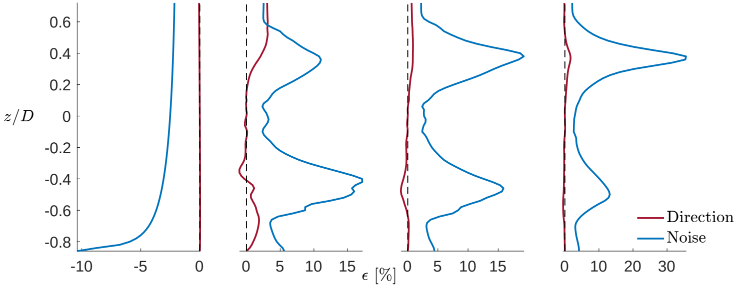
Effect of volume-averaging



Time-averaged difference in the measured line-of-sight to point-like velocities along z

Comparison of errors





Conclusion

Numerical lidar

- Simple integration into existing numerical frameworks
- Implementation is necessary as volume-averaging is function of flow-field and lidar design and location
- Significant volume-averaging effect at wake edges
- Noise error should also be considered

Acknowledgements

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RESEARCH, TECHNOLOGY & GROWTH

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Ingenuity for life

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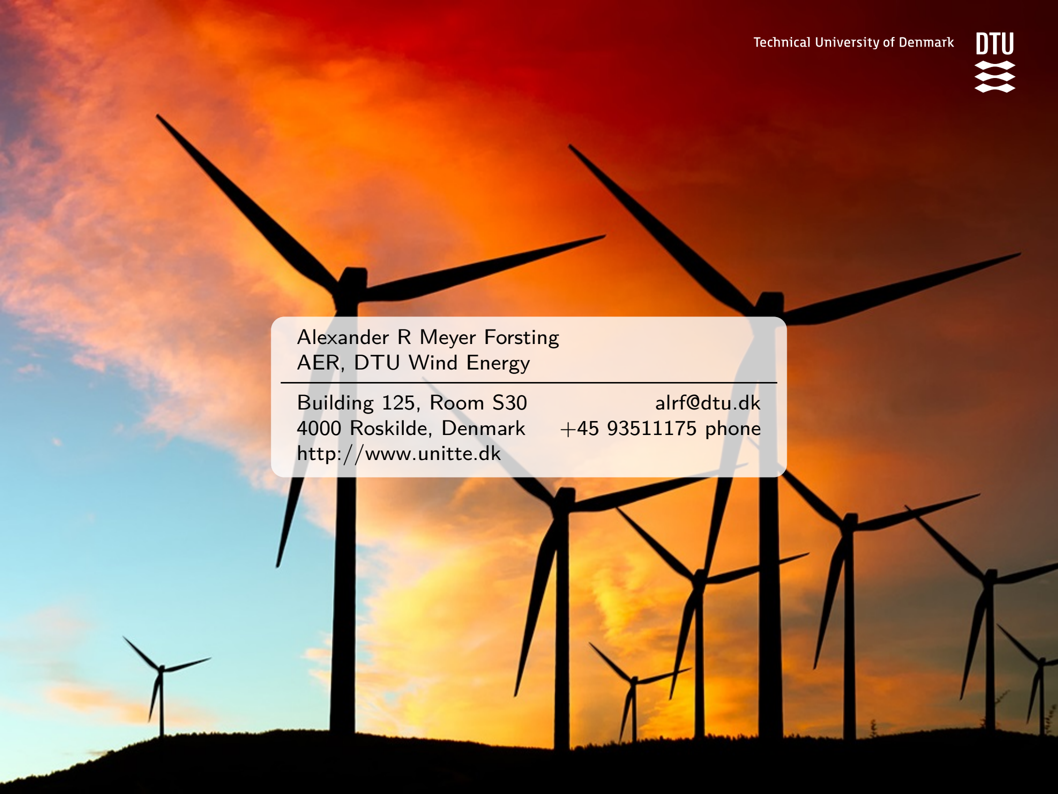
Lidar Technology

ZephIR Lidar



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